

Rosanoff

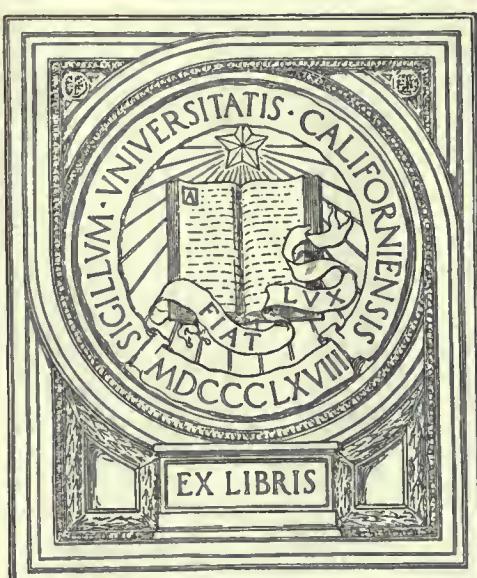
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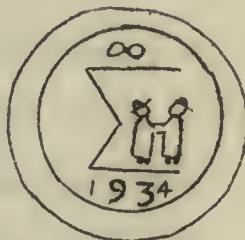
QA275 R6



A PRACTICAL SIMPLIFICATION OF THE METHOD OF LEAST SQUARES

by

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Long Island University  
Galois Institute of Mathematics

A Lecture given at the

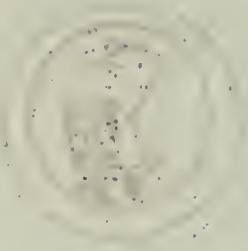
Galois Institute of Mathematics

at

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A PRACTICAL SIMPLIFICATION OF THE METHOD OF LEAST SQUARES.

BY M. A. ROSANOFF, Sc.D., PROFESSOR OF CHEMICAL RESEARCH  
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In course of our researches on partial vapor pressures and the theory of distillation, + my students and myself had frequent occasion to use the Method of Least Squares. To facilitate the extensive computations involved, I devised a simplification and calculated a number of auxiliary formulae, which may save much superfluous labor to others and are therefore reproduced in the following pages.

In the mathematical treatment of scientific results labor is often wasted on a degree of precision in excess of the accuracy of the results themselves. For instance, two experimental figures, 6.7893 and 3.4578, involving an error of at least 1 part in 35,000, might be multiplied with arithmetical rigor to obtain the product 23.47604154 implying an error of 1 part in two billion. An observer of experience, aiming merely to keep his mathematics within the limits of his experimental errors, would multiply the two figures in some such way as this:

$$\begin{array}{r} 6.7893 \\ 3.4578 \\ \hline 20.3679 \\ 2 \ 7157 \\ 3395 \\ 475 \\ \hline 54 \\ \hline 23.4760 \end{array}$$

In the product, written 23.476, the multiplication error of 4 in two million

\*Communicated by the Author.

+Partly summarized in Sydnéy Young's Distillation (Macmillan & Co., London and New York, 1922.)

and the members of the family are accompanying the deceased. The deceased himself is seated in a chair in the middle of the room, his hands clasped in his lap. The mourners are seated in a circle around the deceased, their heads bowed in grief. The atmosphere is somber and quiet, with only the sound of weeping and the occasional whisper breaking the silence.

卷之三

从1950年到1953年，中国在抗美援朝战争中，共毙伤俘敌军100多万人。

-2-

would be negligible compared with the experimental error of the multiplier.

In the use of the Least Squares this type of simplification must not be employed without alert watchfulness, matters being complicated by the additions and subtractions, by which the relative errors are liable to be greatly magnified. The semi-graphic procedure recommended below, vaguely analogous in that it too aims merely to keep the mathematical errors within those of the experiments to be represented, will be found accurate enough for all ordinary purposes and safe. The procedure is based on the substitution of carefully interpolated figures for the actual results of observation.

The given experimental results are plotted on accurately ruled millimeter paper, the scale large enough to show the likely errors of observation or experiment. A smooth curve is drawn free-hand to represent the trend of the points as closely as the eye will allow; or else a neat wavy curve is drawn through the points themselves. In some cases, if the points are more or less evenly thrown by the errors to the one and the other side of any curve on which they might belong, neighboring points may, for the purpose of interpolation, be connected by straight lines. From the curve, or from the broken line, we read the ordinates corresponding to a set of uniformly increasing abscissas to which are assigned the values  $x = 0, 1, 2, 3, 4, \dots, 9, 10$ ; or some similar set. Still further simplification comes of assigning to the abscissas the values  $(x-5) = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ .

These ordinates yield  $\Sigma y$ , and simple further calculation leads to the values of  $\Sigma xy$ ,  $\Sigma x^2y$ , etc. The equations that would ordinarily result have been solved by me in advance for the coefficients  $a$ ,  $b$ ,  $c$ ,  $\dots$  of a series of equations of the form

$$y = a + bx + cx^2 + \dots$$

1997 年 1 月 1 日起施行的《中华人民共和国刑法》第 12 条第 2 款规定：

From the solutions given below, the coefficients  $a, b, c, \dots$  may be obtained immediately by substituting the values of  $\Sigma y, \Sigma xy, \dots$  etc. The final numerical coefficients are yielded by transforming the arbitrary  $x = 1, 2, 3, \dots$ , or  $(x-5) = -5, -4, -3, \dots$ , into the given values of  $x$ .

For the benefit of less experienced computers it may be pointed out that it is a little easier to multiply  $y$  times  $x$ , than  $yx$  times  $x$ ,  $yx^2$  times  $x$ , ... than  $y$  times  $x^2$ ,  $y$  times  $x^3$ , etc.

A simple example will illustrate the procedure recommended and the closeness of its results to those of the direct procedure in general use. For a given set of ten observations, recorded in Table I., let  $y = a + bx$ , and say that the observed values of  $y$  correspond to  $x = 0.5, 1.5, 2.5, \dots, 8.5, 9.5$ .

In general, the Method of Least Squares, applied to a linear relationship, yields the following:

## Formulae for Calculating the Coefficients of $y = a + bx$ , Based on n Observations:

For our given observations these formulae lead to the equation

$$y = 3.0068181 + 1.9936364x \dots \dots \dots \quad (A)$$

In Table I. the first two columns record the observed data; the third gives the values of  $y$  calculated by equation (A); the fourth gives  $\Delta$ , the differences between the calculated and the observed values of  $y$ ,

the 1990s and 2000s, the global media industry is a multi-billion-dollar industry, with major players including the BBC, CNN, and Al Jazeera, and smaller media companies like the New York Times and the Washington Post.

It is possible to use the same method for estimation by just selecting a few points and fitting a curve to them.

which yield the minimum:  $\sum \Delta^2 = 0.168$ . An additional fifth column shows the differences percent.

Table I.

x	y(obs.)	y(calc.)	$\Delta_1$ (calc.-obs.)	$\Delta_1$ , %
0.5	4.10	4.0036363	-0.0963637	-2.35
1.5	6.15	5.9972727	-0.1527273	-2.48
2.5	7.80	7.9909091	+0.1909091	+2.45
3.5	9.85	9.9845455	+0.1345455	+1.37
4.5	12.10	11.9781819	-0.1218181	-1.01
5.5	13.80	13.9718183	+0.1718183	+1.25
6.5	15.90	15.9654547	+0.0654547	+0.41
7.5	18.05	17.9590911	-0.0909089	-0.50
8.5	20.10	19.9527275	-0.1472725	-0.73
9.5	21.90	21.9463639	+0.0463639	+0.21

We now employ the indirect procedure. The observations are plotted on a scale where 50 mm. represent one unit of x, and 20mm. one unit of y. Neighboring points are connected by straight lines. The arbitrary values  $x = 1, 2, 3, \dots, 8, 9$ , and the corresponding values of y read from the broken line are given in the first two columns of Table II. From these we get  $\sum y = 116.90$  and  $\sum xy = 704.35$ , which yield immediately the coefficients a, b, of the required equation by substitution in the following formulae:

Formulae for Calculating the Coefficients of  $y = a + bx$ , Based on Nine Points:

$x = 1, 2, 3, \dots, 8, 9$ .



$$a = \frac{+95 \sum y - 15 \sum xy}{180}$$

$$b = \frac{-15 \sum y - 3 \sum xy}{180}$$

..... (2)

We thus obtain the equation:

Table II

x	y	x(obs.)	y(obs.)	y(calc.)	$\Delta_2$ (calc. - obs.)	$\Delta_2$ %
1	5.15	0.5	4.10	4.0001389	-0.0998611	-2.41
2	7.00	1.5	6.15	5.9976389	-0.1523611	-2.48
3	8.85	2.5	7.80	7.9951389	+0.1951389	+2.50
4	11.00	3.5	9.85	9.9926382	+0.1426389	+1.45
5	12.95	4.5	12.10	11.9901389	-0.1098611	-0.91
6	14.85	5.5	13.80	13.9876389	+0.1876389	+1.36
7	17.00	6.5	15.90	15.9851389	+0.0851389	+0.54
8	19.10	7.5	18.05	17.9826389	-0.0673611	-0.37
9	21.00	8.5	20.10	19.9801389	-0.1198611	-0.60
		9.5	21.90	21.9776389	+0.0776389	+0.35

The third and fourth columns of Table II, reproduce again for comparison the "observed" values of  $x$  and  $y$ ; the fifth gives the values of  $y$  calculated by equation (B); the sixth shows  $\Delta_2$ , the differences between these calculations and the observations and, again, the last column shows the differences percent.

《詩經》歌頌了周朝的豐功偉業，歌頌了周朝的社會制度，歌頌了周朝的道德風氣。

256 *W. H. Dugay*

(4) 2010-2011 年度の実績と 2012-2013 年度の予算

江都縣志

Plainly, the indirect procedure and equation (B) reproduce the results all but as well as the usual direct procedure and equation (A). The sum of the squares of the differences between the calculated and the observed values,  $\sum \Delta_2^2 = 0.171$ , is very close to the minimum,  $\sum \Delta_1^2 = 0.168$ , of the direct procedure.

In place of the nine-point formulae (1), the following based on eleven points, will be found more convenient in some cases:

Formulae for Calculating the Coefficients of  $y = a + bx$ , Based on Eleven Points:  $x = 0, 1, 2, 3, \dots, 8, 9, 10.$

$$\left. \begin{array}{l} a = \frac{35 \sum y - 5 \sum xy}{110} \\ b = \frac{-5 \sum y + \sum xy}{110} \end{array} \right\} \dots \dots \dots \quad (3)$$

Applying these formulae to our test case, we obtain the equation:

Table III. shows the results. The first two columns reproduce once more the "observed" values of  $x$  and  $y$ . The third gives the values of  $y$  calculated by equation (C). The fourth and fifth show the differences between the calculated and the observed values.

Here the sum of the squares of the differences,  $\sum \Delta_3^2 = 0.169$ , is even closer to the minimum  $\sum \Delta_1^2 = 0.168$ , yielded directly by the observations.

The differences between the values of  $y$  from our indirectly gotten equations (B) and (C) and those from equation (A) based immediately on the observations, are small compared with the errors of the observations themselves.

1930, 1931 and 1932,  $\Delta T = 1.5^\circ$

so that we can also get a better understanding of the role of the different mechanisms that form the basis of the protein corona.

2016-01-18 16:00:00

(6) *Scutellaria* *lanceolata* *L.* *var.* *lanceolata*

Self-indulgence and pride are two of the cardinal mortal sins.

(10) *an account of the various methods of finding the area of a circle.*

cautiously because we have not evidence and facts and when  
we do we should say this is owing to ignorance and when we

(3) collapse of *Salinaria* to *Salicornia* by *Salicornia* and *Salinaria* in the same area.

at  $\sin \theta = \frac{1}{2}$  (which is to say that the angle  $\theta$  is  $30^\circ$ ) and  $\cos \theta = \frac{\sqrt{3}}{2}$  (which is to say that the angle  $\theta$  is  $60^\circ$ ).

and lead to a minimum with more than 10% lead (10) which could be

Table III.

x	y(obs.)	y(calc.)	$\Delta_3$ (calc. - obs.)	$\Delta_3$ %
0.5	4.10	4.00272727	-0.09727273	-2.37
1.5	6.15	5.99909091	-0.15090909	-2.45
2.5	7.80	7.99545455	+0.19545455	+2.51
3.5	9.85	9.99181818	+0.14181818	+1.44
4.5	12.10	11.98818182	-0.11181818	-0.92
5.5	13.80	13.98454545	+0.18454545	+1.34
6.5	15.90	15.98090909	+0.08090909	+0.51
7.5	18.05	17.97727273	-0.07272727	-0.40
8.5	20.10	19.97363636	-0.12636364	-0.63
9.5	21.90	21.97000000	+0.07000000	+0.32

Below are given several sets of formulae for calculating the coefficients of parabolic equations of the second, third, and fourth degrees, which will suffice to meet most ordinary needs. In using these formulae the number of significant figures in the products involved should only be reduced with great caution (if at all).

Formulae for Calculating the Coefficients of  $y = a + bx + cx^2$ , Based on Nine Points:  $x = 0, 1, 2, 3, \dots, 7, 8$ .

$$\left. \begin{array}{l} a = \frac{+3052\sum y - 1428\sum xy + 140\sum x^2y}{4620} \\ b = \frac{-1428\sum y + 1037\sum xy - 120\sum x^2y}{4620} \\ c = \frac{+140\sum y - 120\sum xy + 15\sum x^2y}{4620} \end{array} \right\} \dots\dots\dots (4)$$

If the parabola is to pass through the origin, then

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70,00	00000000,00	00000000,00	00,00
65,00	00000000,00	00000000,00	00,00
60,00	00000000,00	00000000,00	00,00
55,00	00000000,00	00000000,00	00,00
50,00	00000000,00	00000000,00	00,00
45,00	00000000,00	00000000,00	00,00
40,00	00000000,00	00000000,00	00,00
35,00	00000000,00	00000000,00	00,00
30,00	00000000,00	00000000,00	00,00
25,00	00000000,00	00000000,00	00,00
20,00	00000000,00	00000000,00	00,00
15,00	00000000,00	00000000,00	00,00
10,00	00000000,00	00000000,00	00,00
5,00	00000000,00	00000000,00	00,00
0,00	00000000,00	00000000,00	00,00

estimated and cannot be often known with any real  
degree of precision, and to estimate it is necessary to make  
a guess at when traffic does loss of control. If the traffic  
is moving at a steady rate, it is necessary to know the  
time for the vehicles to pass the location of the block  
before the traffic can be estimated.

On basis of 2000 vehicles per hour and 1000 vehicles per hour  
the following table is given:

Estimated time for traffic to pass a block

Estimated time for traffic to pass a block

Estimated time for traffic to pass a block

and, taking the present rate of traffic and

Formulae for Calculating the Coefficients of  $y = a + bx + cx^2$ , Based on Eleven Points:  $x = 0, 1, 2, 3, \dots, 8, 9, 10.$

If the parabola is to pass through the origin, then

Formulae for Calculating the Coefficients of  $y = A+B(x-5)+C(x-5)^2 + D(x-5)^3$ , Based on Eleven Points:  $(x-5) = -5, -4, -3, \dots, 3, 4, 5.$

$$\begin{aligned}
 A &= \frac{+6408\sum y - 360\sum y(x-5)^2}{30888} \\
 B &= \frac{+1865\sum y(x-5) + 89\sum y(x-5)^3}{30888} \\
 C &= \frac{-360\sum y + 36\sum y(x-5)^2}{30888} \\
 D &= \frac{-89\sum y(x-5) + 5\sum y(x-5)^3}{30888}
 \end{aligned}
 \quad \left. \right\} \dots \dots \dots \quad (10)$$

$$\left. \begin{array}{l} \text{(7) } \text{minimizing } \text{cost } \text{of } \text{minerals} \\ \text{and } \text{maximizing } \text{minerals} \\ \text{available} \\ \text{and } \text{minimizing } \text{waste} \\ \text{available} \\ \text{and } \text{minimizing } \text{waste} \end{array} \right\} \begin{array}{l} \text{min } \text{cost} = x^T \text{cost} + \text{waste} \\ \text{min } \text{waste} = x^T \text{waste} \end{array}$$

$$\left. \begin{array}{l} \text{(8) } \text{minimizing } \text{cost } \text{of } \text{minerals} \text{ and } \text{maximizing } \text{minerals} \\ \text{available} \\ \text{and } \text{minimizing } \text{waste} \\ \text{available} \\ \text{and } \text{minimizing } \text{waste} \end{array} \right\} \begin{array}{l} \text{min } \text{cost} = x^T \text{cost} + x^T \text{waste} \\ \text{min } \text{waste} = x^T \text{waste} + x^T \text{cost} \end{array}$$

most efficient and economic mining of all minerals and the

$$\left. \begin{array}{l} \text{(9) } \text{minimizing } \text{cost } \text{of } \text{minerals} \\ \text{and } \text{minimizing } \text{waste} \\ \text{available} \\ \text{and } \text{minimizing } \text{waste} \end{array} \right\} \begin{array}{l} \text{min } \text{cost} = x^T \text{cost} \\ \text{min } \text{waste} = x^T \text{waste} \end{array}$$

$x^T \text{cost} \leq (S_{\text{min}}) \text{ cost } \leq \text{minerals available for extraction}$   
 $\leq \text{minerals available for extraction} \leq (S_{\text{min}}) \text{ cost}$   $\leq (S_{\text{min}}) \text{ cost}$

$$\left. \begin{array}{l} \text{(10) } \text{minimizing } \text{cost } \text{of } \text{minerals} \\ \text{and } \text{minimizing } \text{waste} \\ \text{available} \\ \text{and } \text{minimizing } \text{waste} \end{array} \right\} \begin{array}{l} \text{min } \text{cost} = x^T \text{cost} + x^T \text{waste} \\ \text{min } \text{waste} = x^T \text{waste} + x^T \text{cost} \\ \text{min } \text{cost} \leq (S_{\text{min}}) \text{ cost} \leq \text{minerals available for extraction} \\ \leq \text{minerals available for extraction} \leq (S_{\text{min}}) \text{ cost} \end{array}$$

The coefficients of  $y = a + bx + cx^2 + dx^3$  may be obtained directly by the following set of formulae; it will be noted that these still involve  $(x-5) = -5, -4, -3, \dots, 4, 5$ , and not  $x = 0, 1, 2, 3, \dots, 10$ .

Formulae for Calculating the Coefficients of  $y = a + bx + cx^2 + dx^3$  Based on Eleven Points:  $x = 0, 1, 2, 3, \dots, 7, 9, 10$

$$a = \frac{-2592 \sum y + 1800 \sum y(x-5) + 540 \sum y(x-5)^2 - 180 \sum y(x-5)^3}{30888}$$

$$b = \frac{+3600 \sum y - 4810 \sum y(x-5) - 360 \sum y(x-5)^2 + 286 \sum y(x-5)^3}{30888}$$

$$c = \frac{-360 \sum y + 1335 \sum y(x-5) + 36 \sum y(x-5)^2 - 75 \sum y(x-5)^3}{30888}$$

$$d = \frac{-89 \sum y(x-5) + 5 \sum y(x-5)^3}{30888}$$

... (11)

As a rule, it will be simpler to use, not Formulae (11), but (10), then calculate the coefficients  $a, b, c, d$ , of  $y = a + bx + cx^2 + dx^3$  by:  $a = A - 5B + 25C - 125D$ ;  $b = B - 10C + 75D$ ;  $c = C - 15D$ ;  $d = D$ . If the cubic curve must pass through the origin, then:

$$a = 0$$

$$b = \frac{-2310 \sum y(x-5) + 390 \sum y(x-5)^2 + 36 \sum y(x-5)^3}{30888}$$

$$c = \frac{+1085 \sum y(x-5) - 39 \sum y(x-5)^2 - 50 \sum y(x-5)^3}{30888}$$

$$d = \frac{-89 \sum y(x-5) + 5 \sum y(x-5)^3}{30888}$$

..... (12)

Or else, still for the eleven points,  $x = 0, 1, 2, 3, \dots, 9, 10$ , the following

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## 3. What is the relationship between the two types of energy?

3.  $\{x \in \mathbb{R}^n : \|\mathbf{A}x - \mathbf{b}\|_2 \leq \epsilon\}$  is convex.

100, (11) different and 200 of which are thin 21 others in  
soft deposit. 22 to 25 are in intercalated and thin others only 26 (11)  
27 to 32 - 3 are intercalated and thin others are 33 to 37  
and 38 to 40 thin and intercalated 38 are thin others 39 to 40

$$\int \frac{3(3x+1) \cdot 3x + 9(3x+1) \sqrt{3x+1} \cdot 3}{3x+1} dx = \frac{9(3x+1)^2 + 27x\sqrt{3x+1}}{3x+1} dx$$

2000-2001-05(0,1) 2000-05(0,1) 2000-05(0,1) 2000-05(0,1)

1991-1992-3000-100

On the 20th of April, 1865, he was admitted to the hospital, and

$$a = 0$$

$$b = \frac{+36136 \sum y - 11550 \sum yx + 830 \sum yx^2}{51480}$$

.....(13)

$$c = \frac{-11550 \sum y + 4125 \sum yx - 315 \sum yx^2}{51480}$$

$$d = \frac{+830 \sum y - 315 \sum yx + 25 \sum yx^2}{51480}$$

Formulae for Calculating the Coefficients of  $y = A+B(x-5)+C(x-5)^2+D(x-5)^3+E(x-5)^4$ , Based on Nine Points:  $(x-5) = 4, -3, -2, -1, 0, 1, 2, 3, 4$ .

$$A = \frac{+154656 \sum y - 39960 \sum y(x-5)^2 + 1944 \sum y(x-5)^4}{370656}$$

$$B = \frac{+4238 \sum y(x-5) - 3068 \sum y(x-5)^3}{370656}$$

$$C = \frac{-39960 \sum y + 18207 \sum y(x-5)^2 - 1035 \sum y(x-5)^4}{370656}$$

....(14)

$$D = \frac{-3068 \sum y(x-5) + 260 \sum y(x-5)^3}{370656}$$

$$E = \frac{+1944 \sum y - 1035 \sum y(x-5)^2 + 63 \sum y(x-5)^4}{370656}$$

Formulae for Calculating the Coefficients of  $y = A+B(x-5)+C(x-5)^2+D(x-5)^3+E(x-5)^4$ , Based on Eleven Points:  $(x-5) = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ .

For  $\overline{AB} = \overline{CD} = \overline{EFG}$   $\Rightarrow$   $\overline{AB} \cong \overline{CD} \cong \overline{EFG}$

# 卷之三

11. *Constitutive and regulatory genes in the *hsp70* operon*

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10. *Leucanthemum vulgare* L. (Lam.)

the following are the 4  $(4 \times 4)$  square numbers, and the 4  $(4 \times 4)$  square numbers.

$$A = \frac{+41184 \sum y - 6840 \sum y(x-5)^2 + 216 \sum y(x-5)^4}{123552}$$

$$B = \frac{+7460 \sum y(x-5) - 356 \sum y(x-5)^3}{123552}$$

$$C = \frac{-6840 \sum y + 2019 \sum y(x-5)^2 - 75 \sum y(x-5)^4}{123552} \quad \dots \dots \dots (15)$$

$$D = \frac{-356 \sum y(x-5) + 20 \sum y(x-5)^3}{123552}$$

$$E = \frac{+216 \sum y - 75 \sum y(x-5)^2 + 3 \sum y(x-5)^4}{123552}$$

Formulae for Calculating the Coefficients of  $y = a + bx + cx^2 + dx^3 + ex^4$ ,

Based on Eleven Points:  $x = 0, 1, 2, 3, \dots, 8, 9, 10$ .

$$a = \frac{+5184 \sum y + 7200 \sum y(x-5) - 3240 \sum y(x-5)^2 - 720 \sum y(x-5)^3 + 216 \sum y(x-5)^4}{123552}$$

$$b = \frac{-39600 \sum y - 19240 \sum y(x-5) + 17310 \sum y(x-5)^2 + 1144 \sum y(x-5)^3 - 750 \sum y(x-5)^4}{123552}$$

$$c = \frac{+25560 \sum y + 5340 \sum y(x-5) - 9231 \sum y(x-5)^2 - 300 \sum y(x-5)^3 + 375 \sum y(x-5)^4}{123552} \quad 16$$

$$d = \frac{-4320 \sum y - 356 \sum y(x-5) + 1500 \sum y(x-5)^2 + 20 \sum y(x-5)^3 - 60 \sum y(x-5)^4}{123552}$$

$$e = \frac{+216 \sum y - 75 \sum y(x-5)^2 + 3 \sum y(x-5)^4}{123552}$$

If this curve must pass through the origin, then:

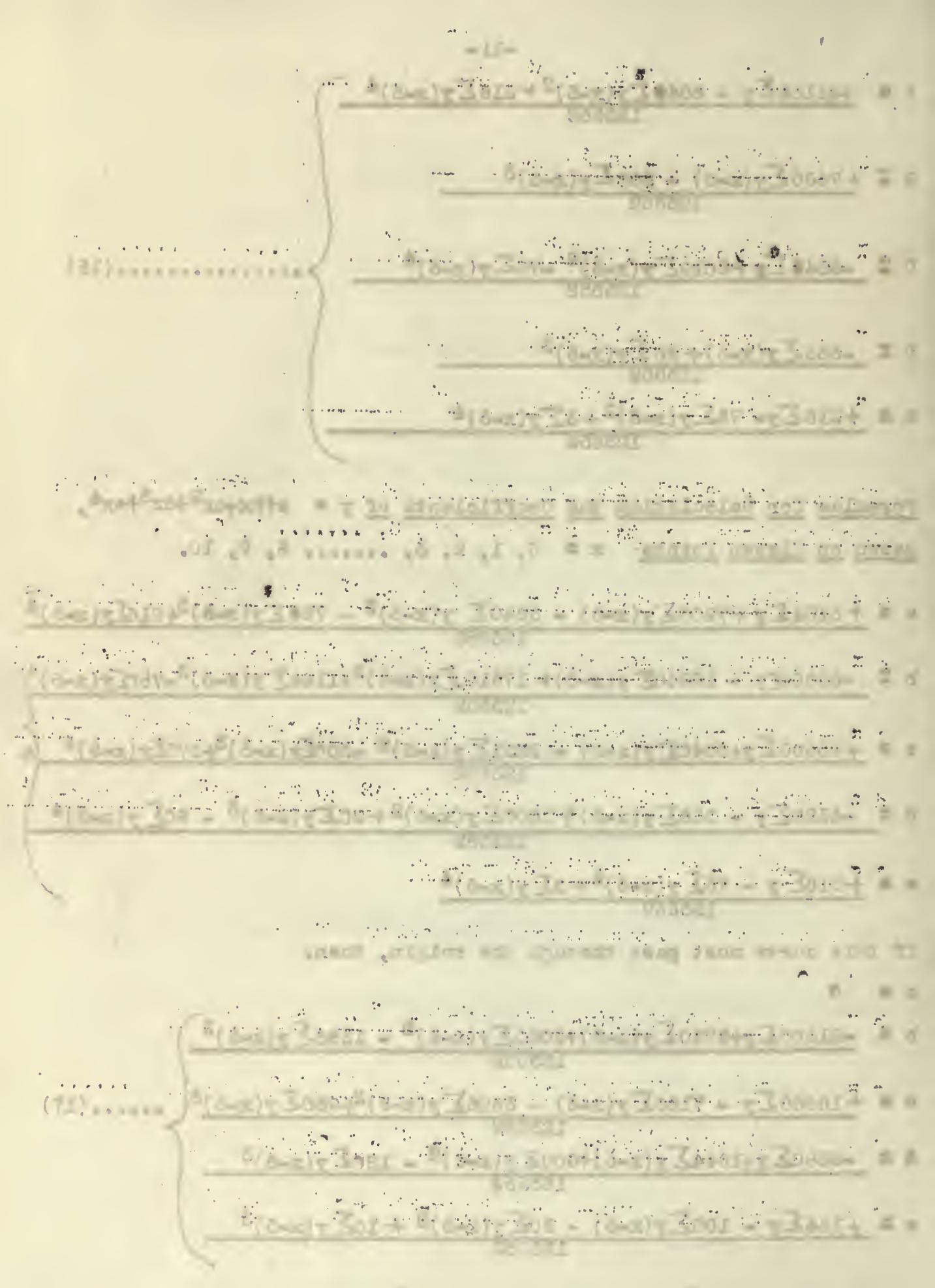
$$a = 0$$

$$b = \frac{-21600 \sum y + 5760 \sum y(x-5) + 6060 \sum y(x-5)^2 - 1356 \sum y(x-5)^3}{123552}$$

$$c = \frac{+16560 \sum y - 7160 \sum y(x-5) - 3606 \sum y(x-5)^2 + 950 \sum y(x-5)^3}{123552} \quad \dots \dots \dots (17)$$

$$d = \frac{-2880 \sum y + 1644 \sum y(x-5) + 600 \sum y(x-5)^2 - 180 \sum y(x-5)^3}{123552}$$

$$e = \frac{+144 \sum y - 100 \sum y(x-5) - 30 \sum y(x-5)^2 + 10 \sum y(x-5)^3}{123552}$$



The saving of labor effected by our procedure will of course grow rapidly with the degree of the equation desired.

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1015

where the main emphasis are to determine which is which and  
which contains which species with the following

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Rb

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